# In-hadron gluon condensate and AdS/LFQCD holography

Stanisław D. Głazek

Institute of Theoretical Physics, Department of Physics, University of Warsaw

A few arguments are put up for a discussion concerning the hope that a systematic application of the renormalization group procedure for effective particles to QCD in the front form of relativistic Hamiltonian dynamics will yield a precise picture of hadrons in which:

**1)** effective quarks are bound by gluons condensed inside hadrons

**2)** in agreement with the expectations suggested by the parton model,

**3)** QCD sum rules,

**4)** models based on AdS/QCD duality in terms of LF QCD holography, and

**5)** the constituent quark model of hadrons.

S. D. Głazek, Acta Phys. Pol. B **42**, 1933-2010 (2011); Few Body Syst. **52**, 367-373 (2012).

stglazek@fuw.edu.pl

# Goal: QCD for hadrons with quality of QED for atoms

$$\mathcal{L}_{QCD} \to \mathcal{H}_{QCD}$$

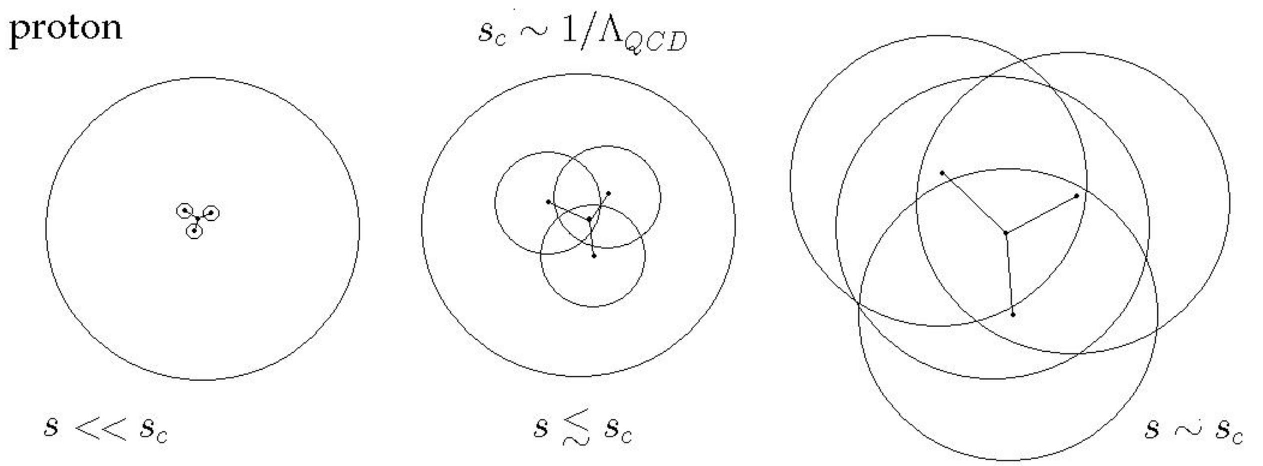
$$H_{QCD} = \int d^3x \, \mathcal{H}_{QCD}$$

$$H_{QCD}|\psi\rangle = E\,|\psi\rangle$$

$$|hadron\rangle = |\psi\rangle$$

$$|\psi\rangle = |qqq\rangle + |qqqg\rangle + |qqqgg\rangle + |qqq\bar{q}q\rangle + ... = \ ?$$

$$|\psi\rangle \sim |q_s q_s q_s G_s\rangle \qquad s = \text{size of effective quark}$$




The RGPEP idea of scale dependence (line = color-transport factor)

Analogy with swarms of colored bees

stglazek@fuw.edu.pl

# **R**enormalization **G**roup **P**rocedure for **E**ffective **P**articles (**RGPEP**)

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canonical front form of Hamiltonian dynamics $t = 0 \to x^+ = 0$

bare canonical theory $\psi(x) \; = \; \int [p] \, q_{0p} \, e^{-ip\,x}$

renormalized (effective) theory $\psi_s(x) \; = \; \int [p] \, q_{sp} \, e^{-ip\,x}$

$s =$ size of effective quark

$$\psi_s \; = \; U_s \, \psi_0 \, U_s^\dagger$$

5th dimension in AdS/QCD $\psi_s(x) = \psi(x, s)$

stglazek@fuw.edu.pl

# Mathematical method

e.g. SDG, Phys. Rev. D **85**, 125018 (2012).

$$\psi_s = U_s \, \psi_0 \, U_s^\dagger \qquad t = s^4 \qquad q_s \equiv q_t \qquad H \equiv P^-$$

$$H_t(q_t) = H_0(q_0)$$

$$H_t(q_0) = U_t^\dagger \, H_0(q_0) \, U_t \qquad \frac{d}{dt} \to \,'$$

$$H_t'(q_0) = [G_t(q_0), H_t(q_0)]$$

$$G_t = -U_t^\dagger U_t' \qquad U_t = T \exp\left( - \int_0^t d\tau \, G_\tau \right)$$

$$G_t = [H_f, H_{Pt}] = \text{generator of RGPEP}$$

stglazek@fuw.edu.pl

# Generator and result

.

$$G_t = [H_f, H_{Pt}]$$

$$H_f = \sum_i p_i^- \, q_{0i}^\dagger q_{0i} \qquad\qquad p_i^- = \frac{p_i^{\perp\,2} + m_i^2}{p_i^+}$$

$$H_t(q_0) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \ldots, i_n} c_t(i_1, ..., i_n)\; q_{0i_1}^\dagger \cdots q_{0i_n}$$

$$H_{Pt}(q_0) = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \ldots, i_n} c_t(i_1, ..., i_n)\; \left(\frac{1}{2}\sum_{k=1}^{n} p_{i_k}^+\right)^2 \; q_{0i_1}^\dagger \cdots q_{0i_n}$$

$$\left(\sum_{mn} |H_{Imn}|^2\right)' = -2\sum_{km}(\mathcal{M}_{km}^2 - \mathcal{M}_{mk}^2)^2 |H_{Ikm}|^2 \le 0$$

**Theory** $V = \delta$, 6-dim $\phi^3$, AF in QCD, limit cycles **+ Phen.** $\Upsilon$, $J/\psi$

stglazek@fuw.edu.pl

# Front form → 7 kinematical symmetries including boosts

$$\boxed{\text{IMF} \;\; \leftrightarrow \;\; \text{CMS}}$$

$$p^+ > 0$$

$$|\Omega\rangle = |0\rangle$$

$$\varphi^2 = \langle\Omega|\frac{\alpha_s}{\pi}G^2|\Omega\rangle = ?$$

Cosmology?

stglazek@fuw.edu.pl

# Constituent Quark Model hypothesis in QCD

$s_c \sim 1/\Lambda_{QCD}$

$$|M\rangle_{s_c} = \sum_{12} \psi_{s_c}(12)\, |12\rangle_{s_c} \qquad |B\rangle_{s_c} = \sum_{123} \psi_{s_c}(123)\, |123\rangle_{s_c}$$

$s \lesssim s_c \qquad W_{ss_c} = U_{s_c} U_s^\dagger$

$$W_{ss_c}\, |12\rangle_{s_c} = |12G\rangle_s \qquad W_{ss_c}\, |123\rangle_{s_c} = |123G\rangle_s$$

$$|M\rangle_s = \sum_{12G} \psi_s(12G)\, |12G\rangle_s \qquad |B\rangle_s = \sum_{123G} \psi_s(123G)\, |123G\rangle_s$$

stglazek@fuw.edu.pl

## Effective eigenvalue problem $\quad H_s|\psi\rangle \;=\; E\,|\psi\rangle$

$$ {}_s\langle 12G|H_s|M\rangle_s \;=\; E_M\,\psi_s(12G) $$

$$ {}_s\langle 123G|H_s|B\rangle_s \;=\; E_B\,\psi_s(123G) $$

$$\boxed{H_s = \frac{\mathcal{M}^2+P^{\perp\,2}}{P^+} \qquad \text{gauge symmetry} \qquad -i\vec{\nabla} \;\to\; -i\vec{\nabla} - g_s\vec{A}}$$

1) the Schwinger gauge $\quad A^\mu = \frac{1}{2}\,(x - x_G)_\nu G^{\nu\mu} + \ldots$

2) color-transport factors $\quad T_i = e^{-ig\int_{\underline{x}}^{x_i} dx_\mu A^\mu}$

3) crude mean field approximation (Abelian) $\qquad$ Głazek-Schaden 1987

$$ \langle G|g_s^2\vec{A}^{\,2}(\vec{x}\,)|G\rangle \sim \frac{1}{4}\langle G|g_s^2 G^{\mu\nu 2}|G\rangle\,(\vec{x} - \vec{x}_G)^2 \quad \to \quad \varphi^2\,(\vec{x} - \vec{x}_G)^2 $$

gauge symmetry restores translational symmetry $\to$ $\vec{x}_G$ drops out

stglazek@fuw.edu.pl

# Front form mesons

$$\mathcal{M}^2_{q\bar{q}} = 4m^2 + 4\left[\vec{k}^{\,2} + \frac{1}{2}m^2\left(\frac{\pi\varphi}{3m}\right)^2 \frac{1}{2}\left(i\,\frac{\partial}{\partial\vec{k}}\right)^2\right]$$

$$k^\perp = \frac{\kappa^\perp}{2\sqrt{x(1-x)}}$$

$$k^z = \frac{2x-1}{2\sqrt{x(1-x)}}\,m$$

$$P = p_1 + p_2\,, \quad x = p_1^+/P^+$$

$$p_1^\perp = xP^\perp + \kappa^\perp\,, \quad p_2^\perp = (1-x)P^\perp - \kappa^\perp$$

stglazek@fuw.edu.pl

# Front form baryons

$$\mathcal{M}_{3q}^2 = 9m^2 + 6\,\vec{K}^{\,2} + \frac{9}{2}\,\vec{Q}^{\,2} - 3m^2\left(\frac{\pi\varphi}{3m}\right)^2 \frac{5}{8}(\Delta_K^2/2 + 2\Delta_Q/3)$$

$$x_i = p_i^+/P^+\,, \qquad p_3^\perp = x_3 P^\perp + q^\perp\,,$$

$$p_2^\perp = x_2 P^\perp - \frac{x_2}{1-x_3}\,q^\perp - \kappa^\perp\,, \quad p_1^\perp = x_1 P^\perp - \frac{x_1}{1-x_3}\,q^\perp + \kappa^\perp$$

$$Q^\perp = \sqrt{\frac{2}{9x_3(1-x_3)}}\;q^\perp\,, \quad Q^z = \sqrt{\frac{2}{9x_3(1-x_3)}}\;(2x_3 - x_1 - x_2)\,m$$

$$K^\perp = \sqrt{\frac{1-x_3}{6x_1x_2}}\;\kappa^\perp\,, \quad K^z = \sqrt{\frac{1-x_3}{6x_1x_2}}\;\frac{x_1-x_2}{1-x_3}\,m$$

## Right values for CQM

$$\omega_M = \frac{\pi\varphi}{3m} , \qquad \omega_B = \sqrt{\frac{5}{8}}\, \omega_M$$

$$\varphi^2_{vacuum} = \langle\Omega|(\alpha_s/\pi)G^{\mu\nu c}G^c_{\mu\nu}|\Omega\rangle \leftrightarrow \varphi^2 = \frac{\langle G|(\alpha_s/\pi)G^{\mu\nu c}G^c_{\mu\nu}|G\rangle}{\langle G|G\rangle}$$

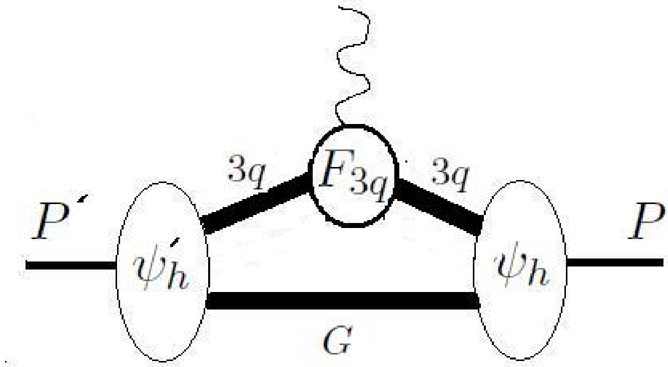
## Front form wave functions for constituent quarks

$$\psi_{qH} = N\, \exp\left\{ -\frac{1}{2n_H m\omega_H} \left[ \left( \sum_{i=1}^{n_H} p_i \right)^2 - (n_H m)^2 \right] \right\}$$

$s \sim 1/\Lambda_{QCD}$ , $n_M = 2$ , $n_B = 3$

stglazek@fuw.edu.pl

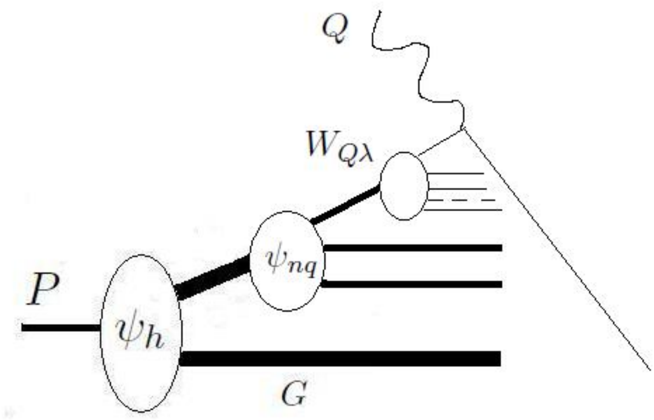
# Hadron form factors at small $Q^2$




$$F_{hadron}(Q^2) \quad \sim \quad e^{\frac{1-<x_q>}{<x_q>} Q^2/(2n_H m\omega_H)} \quad F_{3q}(Q^2) \qquad \text{phenomenology?}$$

stglazek@fuw.edu.pl

# Hadron structure functions




$W_{Q\lambda} \leftrightarrow DGLAP$ phenomenology?

# Comment on AdS/QCD

right variable for Brodsky-Teramond holography $\quad k^{\perp} = \frac{\kappa^{\perp}}{2\sqrt{x(1-x)}}$

Hypotheses:

1) AdS 5th dimension $\leftrightarrow$ quark size $s$ in RGPEP ?

2) $G$-induced oscillator $\leftrightarrow$ soft wall models in IR ?

3) $\kappa_{SW}^2 = 2m\omega_M = \frac{2\pi}{3}\,\varphi_{glue}$ ?

4) AdS/QCD phenomenology $\frac{\kappa_M}{\kappa_B} \sim 1.15 \pm 0.5$ and $\left(\frac{8}{5}\right)^{1/4} \sim 1.125$

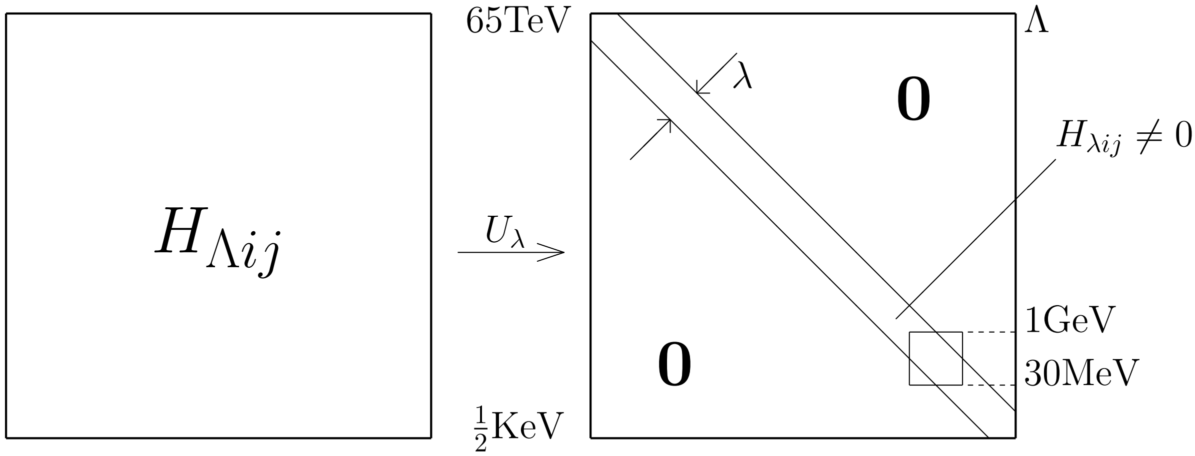
stglazek@fuw.edu.pl

# Conclusion

- Reinterpretation of the gluon condensate
- Constituent phenomenology including the glue component
- Non-perturbative RGPEP method for inspection of QCD
- Boosts and rotations in spectrum: new variables $\vec{k}$, $\vec{K}$, $\vec{Q}$
- Relativistic oscillator (CMS, IMF) and $\boxed{M^2 \sim r^2 \quad \leftrightarrow \quad M \sim r}$
- LF holography and SW AdS model via RGPEP scale parameter (?)

stglazek@fuw.edu.pl

# SRG procedure:




No small denominators in perturbation theory for $H_\lambda$

No explicit dependence of $H_\lambda$ on the eigenvalues, $E$

Eigenvalues $E$ appear on the diagonal when $\lambda \to 0$

## Fock space convergence ?

$$[H^R + CT^R](q_0) \quad = \quad \sum_I \quad c_{0I} \prod_{i \in I} q_{0i}$$

RGPEP form factors $\qquad H_s(q_s) \quad = \quad \sum_I f_s \, c_{sI} \prod_{i \in I} q_{si}$

$$U_s \, q_0 \, U_s^\dagger \quad = \quad q_s$$

quantum fields $\qquad \psi(q_0),\ A(q_0) \quad \to \quad \psi(q_s),\ A(q_s)$

$$|h\rangle \ = \sum_I \phi_{h0}(I) \prod_{i \in I} q_{0i}^\dagger |0\rangle \quad \to \quad |h\rangle \ = \sum_I \phi_{hs}(I) \prod_{i \in I} q_{si}^\dagger |0\rangle$$

$$U_{s_1}\,, \quad U_{s_2}\,, \quad W_{s_2 s_1} \ = \ U_{s_2} U_{s_1}^\dagger$$

$$q_{s_2} \quad = \quad W_{s_2 s_1} \, q_{s_1} \, W_{s_2 s_1}^\dagger$$

equations for scale-evolution in $s$

# Time-honored problem:

$$
\begin{array}{rcl}
H & \rightarrow & H^{\Delta} + CT^{\Delta} \\
H^{\Delta} + CT^{\Delta} & \rightarrow \; RG \; \rightarrow & H_{\lambda} \\
H_{\lambda} & = & ? \\
g_{\lambda} & \sim & \dfrac{1}{\ln \frac{\lambda}{\Lambda_{QCD}}} \\
|\Omega\rangle & = & ?
\end{array}
$$

# History:

**RG** → **SRG** → **RGPEP** → **PT + NPT**

Wilson 1964 — Głazek-Wilson 1993 — Acta Phys. Polon. 1998 . . . — 2011

Wegner 1994

stglazek@fuw.edu.pl